

if the severe cyclones of the transitional periods of the monsoons arise in the way I have above indicated, the milder but more lasting "cyclonic storms" of July and August which are generated in the extreme north of the bay, and which often traverse a great part or the whole of Central and North-Western India before they break up, may nevertheless be formed in the same manner as those of the temperate zone. But this seems to me extremely improbable. With the single exception of the place of their origin, the circumstances of their formation are essentially identical with those of the former class. Moreover, the track which they almost invariably follow seems to be determined by the distribution of the monsoon currents, being along the trough of low pressure which lies between the easterly and westerly branches of the monsoon of Northern India. Although the belt of broken hilly ground running across Central India is generally traversed by the storm vortex, the winds which mainly feed it from the bay have a clear sweep up the great Gangetic plain, and those from the Bombay coast, after surmounting the ghats, have a tolerably unimpeded course across the Deccan plateau, whereas in such cases as the Backerganj cyclone, the whole broad range of the Arakan Yoma, from 5000 to 7000 feet in height, presents an obstacle to the single feeding current from the Bay of Bengal.

I come lastly to the crucial test of temperature—to the question, namely, whether the mean temperature of the air-column in a tropical cyclone is such as to render it specifically lighter than the surrounding atmosphere, and therefore such as to promote an ascending movement. We have indeed in this case no high-level observations to appeal to, such as are furnished to Dr. Hann by the Alpine and other mountain observatories for the storms of the temperate zone. But Dr. Hann has made a rough computation which enables us to bring it fairly to the test, and which in the case of European storms was found to give a result entirely justified by observation. In a paper published in the September number of the *Meteorologische Zeitschrift* he has computed the temperatures of the air-column over a tropical cyclone at different elevations, on the assumption of adiabatic cooling, and has compared these with the average normal temperatures of the atmosphere at the same elevations as deduced from the observations of Newera Eliya, Dodabetta, and Antisana. As the result, he finds that the mean temperature of the former is probably about 2° C. higher than that of the latter, and therefore such as to produce an upward movement of the cyclonic atmosphere, with an acceleration equal to about $\frac{1}{3100}$ of the force of gravity. It may indeed be somewhat greater than this, since in his computation Dr. Hann has assumed a temperature of 28° C. or 82°·4 F., and a relative humidity of only 80 per cent. for the atmosphere of the cyclone at sea-level. In point of fact, observation shows that in a cyclone in the south of the bay the temperature at the sea-level is 79° or 80°, but that the air is saturated, or close upon saturation. Making this correction of the data, the mean temperature of the cyclonic air-column will be about 3° C. higher than that of a normal atmosphere, equivalent to an accelerating force of $\frac{1}{2100}$ of gravity.

From every point of view, then, whether we regard the place and circumstances of their origin, their behaviour after formation, their physical constitution, or the relative activity of the causes supposed to be concerned in their production, the conclusion seems irresistible that tropical cyclones originate in a manner quite different from that ascribed to the storms of the temperate zone; that they are in their early stages a disturbance of the lower atmosphere; and that the primary impulse is given by the ascent and condensation of vapour.

These remarks apply only to the "cyclones" of the beginning and end of the summer monsoon, and the

"cyclonic storms" of the summer months. The storms that traverse Northern India in the winter and early spring, which always travel eastward, and but very rarely descend within the tropic, are of quite a different character, and may not improbably originate in the manner suggested by Prof. Hann.

HENRY F. BLANFORD.

THE DE MORGAN MEDAL.¹

IN 1869 Lord Rayleigh commenced the long series of papers and memoirs in Mixed Mathematics, which the Council had in view in making the award, with an article (*Philosophical Magazine*, vol. xxxviii., third series) "On some Electro-magnetic Phenomena considered in connexion with the Dynamical Theory," founded on Clerk-Maxwell's celebrated "Dynamical Theory of the Electro-magnetic Field" (*Phil. Trans.*, 1865), the subject being "Examples of Electro-magnetic Problems illustrated by comparison with their Mechanical Analogues." I may add, to complete the key-note, thus struck, of Lord Rayleigh's scientific career, that these theoretical results were followed up in the next year by an account of "An Electro-magnetic Experiment," viz. the magnetizing effect of an induced current as dependent on the self- and mutual inductions of the circuits—an early instance of the author's practice of making theory and experiment, or concrete example, illustrate one another. This combination of experimental with mathematical skill and fertility of resource has been conspicuous in Lord Rayleigh's later memoirs on the "Determination of the Ohm and B.A. Unit of Resistance in Absolute Measure" (*Roy. Soc. Proc.*, vol. xxxiv., and *Phil. Trans.*, vol. clxxiii., 1882.)

Confining myself to those earlier memoirs and papers by Lord Rayleigh in which mathematical investigation predominates, the next which calls for notice is one of considerable length, "On $\int_0^1 Q_n Q_{n'} d\mu$ ($Q_n Q_{n'}$ being Laplace's coefficients of orders n, n'), with an Application to the Theory of Radiation" (*Phil. Trans.*, 1870); in which, instead of the two Q 's being multiplied together and integration then effected, the flank of the difficulty is ably turned and the object attained with comparative ease. The mathematical results are illustrated by their application to the problem of "finding the stationary condition when a uniform sphere is exposed to radiation from infinitely distant surrounding bodies."

"A Memoir on the Theory of Resonance" (*Phil. Trans.*, 1871), is the first of Lord Rayleigh's numerous essays in the mathematical theory of sound, which were preliminary, or have been a sequel, to his well-known Treatise, of which the two volumes appeared successively in 1877 and 1878.

The subject of the Memoir is an investigation of air-waves (of the fundamental note) in hollow spaces, whose three dimensions are small in comparison with the wavelength, and which communicate with the atmosphere by necks similarly limited. The theory is shown to be applicable to cases of multiple resonance, as when two or more such hollow spaces are connected by necks and communicate with the external air by necks.

The question of the calculation of c , a factor or coefficient occurring in some of the differential equations, is treated in a noteworthy manner, suggested by the remark that, if the air were replaced by uniform conducting matter of unit specific conducting power, and the sides of the vessel by insulators, c would be the measure of electric conductivity between the interior of the vessel and external space, "an analogy freely availed of, . . .

¹ Address to the London Mathematical Society on the occasion of the presentation of the De Morgan Medal to Lord Rayleigh, November 13, 1890, by the President, J. J. Walker, F.R.S.

much circumlocution being thereby avoided on account of the greater completeness of electrical phraseology."

Having joined our Society in 1871, Lord Rayleigh commenced that series of communications in Applied Mathematics, by which our Proceedings have almost every year since been enriched, with a paper in Physical Optics, "On Verdet's Explanation of Coronas" (vol. iii. p. 267), in which a fallacy of reasoning in his mathematical work is pointed out, but its disappearance in the result explained.¹

This is the first of many important papers evincing the critical manner in which Lord Rayleigh studies the work of his predecessors or contemporaries, while sympathetically and appreciatingly recognizing the value of their contributions to the progress of Physical Science.

To the fourth volume of our Proceedings was contributed an important triad of papers:—

(1) "On the Vibrations of a Gas contained within a Rigid Spherical Envelope," a problem referred to in the Memoir on Resonance, in which the author indicated the only case of the vibration of air within a closed vessel which had prior to that time been completely worked out, the motion being assumed to be irrotational. The solutions of the differential equations are expressed in terms of Harmonic Functions; and, incidentally, interesting theorems in Operations are proved, if in Laplace's Coefficients μ is replaced by the first or second power of the Operative symbol $1 - d/dy$, the Operand being $1/y$.

I would suggest to those having leisure and taste for such a research the possibility of these theorems being generalized for any positive integral power of the Operator.

(2) The second paper, "On the Disturbance produced by a Spherical Obstacle on Waves of Sound," begins with a series of mathematical lemmas of great interest: the expansion of the velocity potential in a harmonic series, with special consideration of the case of its being finite at the poles, and an independent solution of the case of plane waves; a method leading to an expression in terms of Bessel's Functions of fractional order being explained and applied. A comparison of the two results gives a relation of great elegance between a Laplace's Function of order n and a Bessel's Function of order $n + \frac{1}{2}$.

In an Appendix, the expansion of the Velocity Potential for spherical waves is worked out.

Intimately connected with this second of the three papers are two which appeared in the *Philosophical Magazine* for April and June 1871: "On Light from the Sky, its Polarization and Colour," and "On the Scattering of Light by Small Particles," discussed mathematically. The latter paper, setting out with supposing both density and rigidity of the media variable, shows that there can be no direction in the plane perpendicular to the incident ray in which the scattered light vanishes. Hence, either there is no difference of rigidity (the supposition of the former paper), and the vibrations are normal to the plane of polarization; or no difference of density, and the vibrations are parallel to the plane of polarization—which latter alternative is then disproved.²

(3) The third of the triad of papers referred to bears the title, "Some General Theorems relating to Vibrations," with which the Author "had lately become

acquainted during the preparation of a work on Acoustics." The first principle proved is that "the natural periods of a conservative system, vibrating freely about a configuration of stable equilibrium, fulfil the stationary condition."

Some applications of the principle are pointed out, and in illustration of one of these—the approximate calculation of a complicated system, but slightly different from one of a simpler nature—the problem of the transverse motion of a stretched string of slightly unequal longitudinal density is worked out. Another instructive example is that of the tones of a square plate when the type of vibration is such that the nodal lines are central and parallel to the edges, the tone being then gravest, and that in which the diagonals are the nodal lines. The former case is here discussed mathematically for the first time, and Chladni's experimental results shown to be confirmed. An important remark on the possibility of expansion by Fourier's Theorem, Laplace's Series, or Bessel's Functions, in a large number of cases being inferred from physical considerations, is added.

The second principle is that forces which vary as the component velocities (absolute or relative) of the vibratory motions of the parts of the system may be advantageously treated by the method of Virtual Velocities. The investigation leads to the introduction of a function, F (the "Dissipation Function"), which, like the Kinetic and Potential Energy Functions, is a positive quadratic function of the co-ordinates, and represents the rate at which Energy is dissipated. Its application in the fourth chapter of the Author's great work on Sound will at once occur to those acquainted with it. Finally, a law of reciprocity of a very general character is established, relating to the interchangeability of forces and motions of any two types.

The subject of Reciprocal Theorems in Dynamics has been recently (January 12, 1888) treated in our Proceedings by Prof. Lamb, led thereto by a reciprocal theorem, proved by von Helmholtz in a paper "On Least Action" (*Crelle*, vol. c.), which seems much the same as Lord Rayleigh's, if somewhat more generally expressed.

Arising out of this theorem is "A Statical Theorem," communicated by Lord Rayleigh to the *Philosophical Magazine* (vol. xlviii., 1874), referring to a reciprocal property of a system capable of vibrating, with or without dissipation, if displaced from a position of stable equilibrium. The application of the principle in Acoustics (introduced into the "Theory of Sound") formed the subject of a communication to the Royal Society (Proceedings, xxv., 1876).

In this enumeration of some of Lord Rayleigh's earlier papers should be included two of a pure mathematical character on Bessel's Functions, viz. (1) "Notes" thereon, published in the May number of the *Philosophical Magazine*, 1872, in which their great utility in questions of the conduction of heat or electricity, or of a hydrodynamical nature, with conditions to be satisfied by circular, spherical, and cylindrical surfaces, is pointed out. It is needless to remark how constantly these Functions have since been pressed into service in such cases.

(2) "On the Relation between the Functions of Laplace and Bessel," communicated to our Society (Proceedings, vol. ix., January 1878), in which the Function of zero order is shown to be the limit of Legendre's (P_n), and the general Function that of an "Associated" Function of Laplace, where n becomes infinite, but $n \sin \text{angle} \cos \mu$ remains finite. A linear relation among three consecutive Associated Functions, which in the limit becomes the well-known one among three consecutive Bessel's Functions, established by their discoverer, follows.

In connexion with the two papers just noticed, allusion may be made to a "Note," also contributed to our Proceedings (June 11, 1874) "On the Numerical Calculation of the Roots of Fluctuating Functions," pointing out how the difficulty of evaluating them when the argument is neither very small nor great may sometimes be met; the

¹ What regretful thoughts are stirred up by the recollection that Clerk-Maxwell joined in a discussion of this paper, commenced by Sir W. Thomson! From 1871 to his death, Prof. Maxwell was a frequent participator in, as well as contributor to, our meetings. "Quis desiderio . . . !"

² The volumes of the *Philosophical Magazine* for the same year (1871) contain two other optical papers of great interest:—

"On Double Refraction" (vol. xli.), in which the mathematical treatment of the subject, by supposing density to be a function of the direction of vibration, leads to the replacing Fresnel's ellipsoid by its polar reciprocal. Reference is made to Stokes's Report.

"On Reflexion of Light from Transparent Matter" (vol. xlii.), the object of which is "to see how far the facts might be accounted for by the different hypotheses which have been made as to the condition of the æther in transparent matter."

method, though not limited to, having arisen out of, and being exemplified by, Bessel's Functions.

I pass now to Lord Rayleigh's work subsequent to the award of the medal in 1887, which the Council are enjoined by the rules for their guidance to take into special consideration. In the *Philosophical Magazine* for August 1887 there is a paper "On the Maintenance of Vibrations by Forces of Double Frequency," a sequel to that "On Maintained Vibrations," *ibid.*, April 1883. To this subject Lord Rayleigh tells us his attention had been recalled by Mr. Glaisher's Address to the Royal Astronomical Society in the preceding February, in which he had given an interesting account of the treatment of mathematically similar questions, occurring in the Lunar Theory, to those herein considered, by Mr. Hill, published in the *Acta Mathematica* of the preceding year. The appearance of a "dissipative term" in the equation of motion of the vibrating body prevents it being treated as a special case of Hill's similar equation in the motion of the Lunar Perigee. The results are expressed by determinants, and one of the recent papers of Dr. Muir is referred to, on the relations of the special class of determinants involved to continued fractions. Applications to the case of the vibrations of a laminated medium, in which the mechanical properties are periodic functions of one of the co-ordinates—a problem connected with the colours of thin plates; and to the problem of the stationary vibrations of a string of variable density fixed at two points, follow.

In our Proceedings for November 1887, will be found a paper "On the Stability or Instability of certain Fluid Motions," forming a sequel to a former paper on the same subject in our eleventh volume (1880), to which Lord Rayleigh's attention had been re-directed by a recent work of Sir W. Thomson's. In this the subject is treated with greater generality.

In the following year Lord Rayleigh communicated to our June meeting a short but interesting investigation, "On Point, Line, and Plane Sources of Sound," giving rise to a definite integral the connection of which with Bessel's Functions is worked out by Lipschitz's method (*Crelle*, 1859), referred to in the paper of 1877 on Bessel's Functions.

In the Royal Society Proceedings, December 13, will be found a paper in which Mr. Love's objections (*Phil. Trans.*, 1888, A, p. 545), to the line of argument followed in Lord Rayleigh's paper in the thirteenth volume of our Proceedings (1881), "On the Infinitesimal Bending of Surfaces of Revolution," are replied to with great fulness. The interest aroused by these discussions will be well remembered by those who followed them; and allusion to them has since been frequently made at our meetings.

The same memoir by Mr. Love drew forth another mathematical investigation by Lord Rayleigh (*Roy. Soc. Proc.*, February 26, 1889), "On the Free Vibrations of an Infinitely Long Cylindrical Shell."

At our meeting, April 11, 1889, a paper by Lord Rayleigh was read, "On the Free Vibrations of an Infinite Plate of Homogeneous Isotropic Elastic Matter," a particular case of which "On Waves Propagated along the Plane Surface of an Elastic Solid" had been investigated in a communication to our November meeting of the year 1885.

In the before-mentioned years (1888-89), Lord Rayleigh contributed four important papers in the mathematics of Physical Optics to the *Philosophical Magazine*.

(1) "On the Reflexion of Light at a Twin Plane of a Crystal" (*Phil. Mag.*, September 1888). For the calculation of reflexion at the surface between twin crystals, the electric theory of Clerk Maxwell is found to be the only one satisfying the conditions essential to success—as capable of explaining at once Fresnel's laws of double refraction and those governing the intensity of reflexion when light passes from one

isotropic medium to another. Starting with Maxwell's equations connecting the components of the electric displacement and current, magnetic force, electro-magnetic momentum and force, and taking the plane of transition as $x = 0$, Fresnel's tangent formula for isotropic reflexion is obtained; then his law of velocity for propagation in a crystal.

Reflexion at a twin plane is then limited to the cases of the plane of incidence, being (1) coincident with, (2) perpendicular to, the plane of symmetry. In the former case it is shown that the ratio of the amplitudes of the reflected and incident light is equal to the difference of two expressions proved to be equal, whether the vibrations are parallel or perpendicular to the plane of incidence, and no light is reflected whether the incident light be natural or plane-polarized, or elliptically polarized.

The values of the parameters which multiply the exponents in the two reflected waves in the second case are then worked out. Finally, particular cases of these general results are discussed.

(2) "On Interference of Light radiated from Moving Molecules" (*Phil. Mag.*, vol. xxvii. 1889) combats Ebert's conclusions by showing that the maximum admissible retardation is 4.5 times greater than assumed by him (*Wied. Annalen*), whence the width of the spectral lines should be much greater than found, which would involve a blow at the dynamical theory of gases.

(3) "On Complete Radiation at a Given Temperature" (*ibid.*), founded on Gouy's "Theory of Irregular Impulses," investigates what type of similar impulses would by their aggregation represent complete radiation. The probability of various amplitudes depends on principles explained in an article in the *Philosophical Magazine*, August 1880, "On the Resultant of a Large Number of Vibrations of same Pitch and Arbitrary Phase." The form of impulse (familiar in the Theory of Errors)—

$$\phi(x) = \text{exponent}(-c^2x^2),$$

which is resolved into harmonic components by Fourier's Theorem, is discussed.

(4) "On Achromatic Interference Bands" (*Phil. Mag.*, vol. xxviii., August and September 1889). A development of notice of in the article "Wave Theory" (*Enc. Brit.*, 1888), founded on Cauchy's law of dispersion; with a reference to Mascart, "Achromatism of Interference" (*Comptes Rendus*, March 1889).

These abstracts, imperfect as they are, may serve to convey to those who have not closely followed his work some idea of the great variety of subjects in Mixed Mathematics discussed and advanced by Lord Rayleigh, and on which that distinguished reputation in these domains is founded, their recognition of which the Council of the London Mathematical Society desire to mark by the present award of the De Morgan Medal.

A NEW FOSSIL MAMMALIAN FAUNA.

THE trustees of the British Museum have recently added to the collection at South Kensington a large series of fossil remains of mammals purchased from Prof. C. J. Forsyth-Major, which are of especial interest from several distinct points of view. These remains were obtained within the last two or three years by Prof. Forsyth-Major from a Tertiary deposit in Samos—an island in the Turkish Archipelago, lying immediately opposite the town of Ephesus, and to the south-south-west of Smyrna. This deposit, which has been discovered only quite recently, appears to be absolutely full of the bones of mammals; and in this respect it agrees with the contemporary deposits of the celebrated Pikermi ravine near Athens, the wonderful mammalian fauna of which has been fully made known to us by the labours and writings of Prof. Albert Gaudry, of the Paris Museum, and other palæontologists.